Optimal Distribution of Temperature Driving Forces in Low-Temperature Heat Transfer

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A fairly extensive review of research on optimal distribution of driving forces in heat-transfer processes is provided. Four different guidelines for specifying the temperature profiles in heat exchangers have been compared. Not surprisingly, the irreversibilities due to heat transfer in a heat exchanger of given size were found to be minimized when the temperature difference is proportional to the absolute temperature. Comparing a design with an optimal temperature profile and a design with a uniform temperature difference throughout the heat exchanger, sensitivity analyses illustrated that savings in irreversibilities increase with decreasing temperature level and increasing temperature span for the cooling load. Heat exchanger size was found to be of negligible importance. The results indicated that optimal utilization of heat exchanger area is of little importance for processes operating above ambient temperature, while significant savings can be obtained by optimal distribution of temperature driving forces in processes below ambient temperature. *AIChE J*, 61: 2447–2455, 2015
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Introduction

Besides environmental and social aspects, process plants are designed and operated with the purpose of seeking return on investments.[1] Hence, economic criteria are required to evaluate different design options and carry out optimization.[2] In an early phase of process design, it is normally not possible to account for all fixed and variable costs.[2] When comparing the relative merits of different structural options in a flow sheet and parameter settings, the economic criteria can be simplified by neglecting common items.[2]

The thermodynamic performance of a heat exchanger and thereby the energy cost related to operation can be improved by reducing its temperature driving forces. This does, however, require increasing the heat exchanger size and thereby the investment cost. In heat exchanger network synthesis and design, this conflict is often accommodated by assigning a minimum temperature difference in the heat exchangers.[2,3] The minimum temperature difference is then used as an economic trade-off parameter, balancing the operating cost related to energy use and the investment cost of heat exchangers.

This approach has been adopted in many studies for design of cryogenic refrigeration processes such as liquefaction of natural gas.[4] However, as pointed out by Jensen and Skogestad,[4] the use of a minimum temperature difference constraint in optimization of natural gas liquefaction processes leads to nonoptimal utilization of the heat exchanger area. This indicates that a design strategy aiming at uniform temperature difference throughout the heat-transfer process gives nonoptimal distribution of temperature driving forces. This is related to the behavior of exergy of heat, which increases steeply with decreasing temperature below ambient.

Different design guidelines for optimal distribution of temperature driving forces in heat transfer have been proposed in the literature. The objective has been to minimize irreversibilities in a heat exchanger of given size. Previous studies comparing different design guidelines have concluded that the differences in irreversibilities are small. Nevertheless, these studies have focused on heat transfer at relatively high-temperature levels. At low-temperature levels, however, difference in performance of the various design guidelines are expected to be more pronounced.

The scope of this study is to investigate how the temperature driving forces in a heat exchanger of given size should be distributed to minimize the irreversibilities associated with its operation. This has been done by comparing different design guidelines proposed in the literature based on different assumptions. As optimal utilization of heat exchanger area is of special importance in cryogenic processes where refrigeration is expensive, the focus is on low-temperature applications.

First, a brief introduction to exergy of heat is given. This is followed by a discussion on optimal distribution of temperature driving forces in heat exchangers. Based on this, different design strategies are compared for design of a simple heat exchanger. A case study is presented to investigate the influence of the operating conditions of the heat-transfer process, such as the temperature level, the temperature range of the cooling load, and the size of the heat exchanger. Finally, implications with respect to cryogenic processes are briefly discussed.

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Background

Exergy of heat

Exergy is a measure of energy quality indicating the maximum work that can be extracted from a system when brought to equilibrium with its surroundings (or alternatively, the minimum work that must be supplied to bring a system in equilibrium with the surroundings to a given state). A process in which the exergy is conserved is denoted a reversible process. In this case, both the system and its environment can be restored to their initial states without any residual effects in either of them.[5] Irreversibilities are, however, present in all real processes.

The thermodynamic performance of a process can be measured through its exergy efficiency. The rational efficiency[5] of a system is defined as the ratio of exergy output to exergy input

$$\psi = \frac{\sum \Delta\dot{E}_{out}}{\sum \Delta\dot{E}_{in}} = \frac{\sum \Delta\dot{E}_{out}}{\sum \Delta\dot{E}_{out} + \dot{I}} \tag{1}$$

where $\dot{I}$ is the sum of process irreversibilities. For all real processes, the rational efficiency is less than unity, and maximizing the exergy efficiency is equivalent to minimizing the irreversibilities. A review of different exergy efficiency formulations and their applications has been presented by Marmolejo-Correa and Gundersen.[6]

A prerequisite for reversible processes is infinitesimal driving forces. For a heat-transfer process, this is equivalent to an infinitesimal temperature difference between the heat source and a heat sink. As this would give an infinite heat transfer area, a finite temperature difference is required in practical applications. This temperature driving force is a source to entropy production (irreversibilities).

For a given overall heat-transfer coefficient U, the heat exchanger area required to transfer a heat flow $\dot{Q}$ across a temperature difference ΔT is given as

$$A = \frac{\dot{Q}}{U \cdot \Delta T} \tag{2}$$

The exergy of a heat flow $\dot{Q}_i$ available at an absolute temperature T_i is given as

$$\dot{E}(\dot{Q}_i) = \dot{Q}_i \cdot \left(1 - \frac{T_0}{T_i}\right) \tag{3}$$

where T_0 is the ambient temperature.[5] According to the common sign convention used e.g. in mechanical engineering, a heat flow is positive when heat is transferred to a system. Hence, heat transfer to a system would leads to increased exergy above ambient temperature but reduced exergy below ambient temperature.[5] From Eq. 3, the irreversibilities associated with heat transfer between two constant temperature systems can be expressed as

$$\dot{I} = \dot{Q} \cdot T_0 \cdot \left(\frac{1}{T_C} - \frac{1}{T_H}\right) = \dot{Q} \cdot T_0 \cdot \frac{\Delta T}{T_H \cdot (T_H - \Delta T)} \tag{4}$$

where T_H is the temperature of the source and T_C is the temperature of the sink.

In Figure 1, the exergy of heat per unit of heat is plotted as a function of the absolute temperature relative to the ambient temperature. At ambient temperature, a system would be in equilibrium with its surroundings, hence, heat contains no exergy. As the temperature increases above

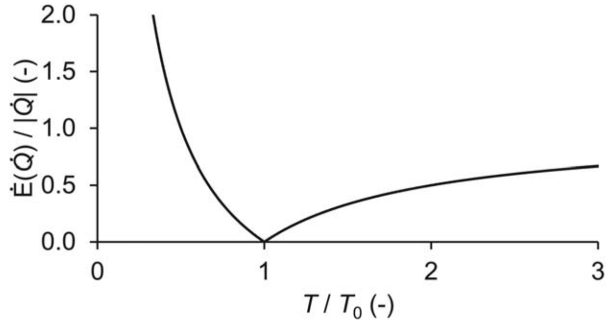

Figure 1. Ratio of the exergy of heat to heat flow as a function of temperature relative to the ambient.

ambient, the exergy of heat increases monotonically, asymptotically approaching unity. For temperatures below ambient, the exergy of heat grows asymptotically toward infinity when the temperature approaches zero. At a temperature equal to half the ambient temperature, the exergy content is equal to the amount of heat. This trend in the exergy of heat indicates that to minimize irreversibilities associated with heat transfer, more attention should be paid to keeping the driving forces small at low-temperature levels. This is also obvious from Eq. (4).

Optimal heat exchanger design

Many studies have been performed on the subject of optimal design and operation of heat exchangers. In this work, the primary focus is on the influence of the driving forces in heat transfer. Over the years, a variety of theories have been proposed for optimal utilization of heat exchanger area, to minimize the entropy production or irreversibilities associated with heat transfer.

Optimal Distribution of Driving Forces. The theory of equipartition of entropy production was introduced by Tondeur and Kvaalen[7] as a framework for optimal configuration of heat- and mass-transfer processes. They found that the total entropy production within a unit operation or process was minimized when the entropy production rate was uniformly distributed in space and time. For the case of constant phenomenological transport coefficients (in the validity range of linear equilibrium thermodynamics and Onsager's relations), this was found to be equivalent to equipartition of driving forces.[7,8]

Later, Sauar et al.[9] proposed the principle of equipartition of forces, as an alternative approach to design optimization. From irreversible thermodynamics, they found exergy loss in coupled transport of heat, mass, or charge to be minimized when the driving forces in the process were uniformly distributed. This was derived for heat-transfer processes with the driving force expressed as $\Delta(1/T) = \left(\frac{1}{T_C} - \frac{1}{T_H}\right)$. The heat flux is then given as

$$\dot{q} = \frac{\dot{Q}}{A} = L \cdot \Delta(1/T) \tag{5}$$

where L is a phenomenological heat-transfer coefficient that depends on the intensive thermodynamic variables of the system but not the driving force.[9] Furthermore, the local entropy production rate per unit of area can then be expressed as

$$\frac{\dot{S}_{\text{prod}}}{A} = L \cdot (\Delta(1/T))^2 \quad (6)$$

The principle of equipartition of forces was derived under the assumption that the transport process is described by independent forces and that the system is linked by the Gibbs–Duhem equation.[9] According to Kjelstrup et al.,[10] the principle of equipartition of forces is a general principle that also holds for cases where the transfer coefficients are not constant. Sauar et al.[9] stated that when L and $\dot{Q}$ are not constant in space and time, a uniform local entropy production rate will not give the minimum total entropy production. Other studies have, however, reached a different conclusion.

According to Xu,[1] the phenomenological heat-transfer coefficient can be approximated by $L \approx U{\cdot}T^2$, where U is the heat-transfer coefficient in the conventional heat-transfer equation. Based on simplified economic considerations, Xu[1] derived that the temperature driving force at any point i in a heat exchanger should be given as

$$\Delta(1/T)_i = \sqrt{\frac{a}{e \cdot T_0 \cdot L_i}} \quad (7)$$

to minimize the sum of capital and operating costs of the heat exchanger. In Eq. 7, a is assumed to be the unit cost of heat-transfer area and e the unit cost of thermal exergy. In this derivation, exergy losses due to pressure drop were omitted. In conventional form, the driving force was approximated as

$$\Delta T_i \approx T_i \cdot \sqrt{\frac{a}{e \cdot T_0 \cdot U_i}} \quad (8)$$

where T_i is the temperature at which the heat is transferred.

For a given heat exchanger area A, Xu[1] found that the optimal heat-transfer driving force could be expressed as

$$\Delta(1/T)_i = \frac{1}{A} \cdot \frac{c}{\sqrt{L_i}} \quad (9)$$

or alternatively as

$$\Delta T_{\text{i}} \approx \frac{T_i}{A} \cdot \frac{c}{\sqrt{U_i}} \quad (10)$$

Here

$$c = \int_0^Q \frac{\mathrm{d}q}{\sqrt{L_i}} \approx \int_0^Q \frac{\mathrm{d}q}{T\sqrt{U_i}} \quad (11)$$

Under the assumption that $T \gg \Delta T$, Chang et al.[11] reached the same conclusion as Xu[1] (Eqs. 10 and 11) when minimizing the entropy generation in a heat exchanger with a given area.

Bejan[12–14] also arrived at the conclusion that the temperature difference between the hot and cold streams in a heat exchanger should be proportional to the absolute temperature, and also noted that this is a well-known design principle in cryogenic engineering.[14] From variational calculus, Balkan[15] deduced equivalence between the theory of equipartition of entropy generation and heat exchanger temperature profiles where the ratio T_H/T_C is constant. This is equivalent to the statements of Xu[1] and Chang et al.[11] that the temperature difference should be proportional to the absolute temperature.

As can be observed from Eqs. 7 and 9, the findings of Xu[1] and Chang et al.[11] imply that the optimal driving force $\Delta(1/T)$ is inversely proportional to the square root of the phenomenological heat-transfer coefficient. This indicates that the driving force $\Delta(1/T)$ would be constant only if L is constant. Xu[1] found the principle of equipartition of forces[9] to be equivalent to a temperature difference ΔT proportional to the square of the temperature and independent of heat-transfer coefficient, which can be deduced assuming $T \gg \Delta T$. Haug-Warberg[16] also raised questions about the principle of equipartition of forces and found that it did not provide a solution of minimum total entropy production in the case of heat transfer with a nonconstant phenomenological heat-transfer coefficient. This was illustrated by an example where the heat transfer was defined in different ways for two sections of a heat exchanger.

In response, Sauar et al.[17] pointed out that the principle of equipartition of forces is based on local linear relations between fluxes and forces in the system. Kjelstrup et al.[18] stated that in order for the theory to hold, the phenomenological heat-transfer coefficient can be an arbitrary function of the temperature and spatial parameters of the heat exchanger, yet it must be independent of the heat flux or the driving force. Furthermore, Kjelstrup et al.[18] found the example presented by Haug-Warberg[16] not to fulfil this requirement. Hence, for this case, the principle does not apply.[18] The deviations from optimality were for this case, however, found to be so small that the principle of equipartition of forces could be a useful rule of thumb also in cases where it does not apply in a strict sense.[18]

With the purpose of verifying that the principle of equipartition of forces does not depend on a constant phenomenological transfer coefficient, Nummedal and Kjelstrup[19] presented examples where the overall heat-transfer coefficient U was assumed constant. However, no comparison was made with other principles for optimal distribution of driving forces. In a later study, Johannessen et al.[20] found that uniform distribution of entropy production led to smaller total entropy production than uniform distribution of driving forces $\Delta(1/T)$. The results did, however, indicate that the principle of equipartition of forces would be likely to estimate the real solution within an error less than one percent in practical applications.[20]

Performance Assessment. Balkan[15] compared the performance of different strategies for distribution of driving forces for design of a heat exchanger with given heat duty and area, assuming a constant overall heat-transfer coefficient U. The principle of equipartition of entropy production was found to give slightly smaller total entropy production than the principle of equipartition of forces, and only a small increase in entropy production was observed for a design with a constant temperature difference ΔT.[15] The results presented by Balkan[15] may, however, be influenced by the fact that the heat transfer took place at relatively high temperature, distributed over a relatively narrow temperature range.

Balkan[15] found that iterations were required to find the proper operating conditions using the principle of equipartition of forces or the principle of equipartition of entropy production. Hence, as only small performance deterioration was observed, Balkan[15] suggested applying a uniform temperature difference (which requires no iterations) as a short-cut design method.

According to Balkan,[21] altering the temperature profiles in a heat exchanger leads to changes in the heat-transfer coefficient of the heat exchanger, which may drastically change entropy production. If the difference in inlet and outlet temperature for a stream in the heat exchanger is reduced, a higher flow rate is required to provide the same heat transfer. With a higher flow rate, the heat-transfer coefficient is improved, especially if fluid resistance dominates the heat-transfer rate.[21] Hence, the temperature driving forces and thereby the entropy production can be reduced. The opposite holds if the temperature range is increased.[21] When altering the design of a two-stream heat exchanger to obtain equipartition of entropy production, assuming that both streams can be changed, Balkan[21] proposed to make the changes for the stream that will experience an increase in the flow rate.

To assess the thermodynamic performance of a heat exchanger, an equipartition factor was defined by Thiel et al.[22] as the ratio of entropy production in a system where the theory of equipartition of entropy is fulfilled to the entropy production in the actual system. Thiel et al.[22] found the potential for exergy efficiency improvement by moving to a system with equipartition of entropy production to be largest in cases where both the equipartition factor and the rational efficiency are low. If a system already has small irreversibilities, the effect of redistributing the temperature driving forces is, of course, small.[22]

In the following, different strategies for distribution of heat-transfer driving forces are compared for design of a simple heat exchanger under different operating conditions, to identify the influence of these design guidelines on the final design. Based on the characteristics of exergy of heat (Figure 1), focus is put on the effect of temperature level.

Simple Counter-Current Heat Exchanger

To examine the influence of the distribution of temperature driving forces on heat-transfer performance, a case study was performed for a simple counter-current heat exchanger model as illustrated in Figure 2. For a given hot stream with a constant heat capacity flow rate $(\dot{m}\cdot c_p)_{\mathrm{H}}$, supply temperature T_{high} and target temperature T_{low}, the cold stream temperature profile was manipulated such that the heat exchanger conductance value (UA) was kept constant. The heat exchanger irreversibilities were then compared for different cold composite curve designs. As the objective was to study the influence of temperature driving forces on heat-transfer irreversibilities, this case study was performed under the assumption of a constant overall heat-transfer coefficient U.

Based on the findings in the preceding sections, four different design guidelines were compared for the cold composite curve:

1. A uniform temperature difference throughout the heat exchanger: $\Delta T = C_{\mathrm{uniform}} = \Delta T_{\min}$.
2. A temperature difference proportional to the temperature of the hot stream: $\Delta T = C_{\mathrm{linear}}\cdot T_{\mathrm{H}}$.
3. A constant value for the difference in the inverse of the temperature: $\Delta(1/T) = C_{\mathrm{inverse}}$.
4. A temperature difference proportional to the square of the temperature of the hot stream: $\Delta T = C_{\mathrm{square}}\cdot T_{\mathrm{H}}^2$.

For the design solutions where the temperature difference is defined as a function of the hot stream temperature, the cold stream temperature could also have been used. For design specification 2, the linear behavior is conserved (with a slight change in the value of the constant). For design specification 4, however, the quadratic form will have a small deviation. One may notice that design specifications 3 and 4 cannot be obtained for a constant value of the heat capacity flow rate for the cold stream $(\dot{m}\cdot c_p)_{\mathrm{C}}$.

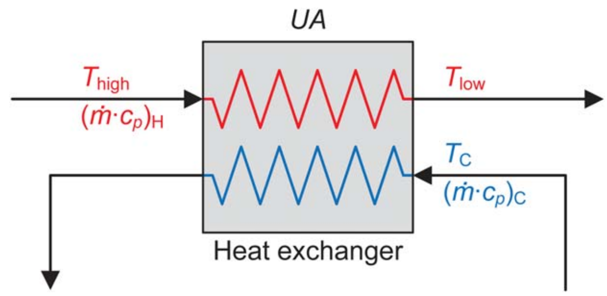

Figure 2. Simple heat exchanger model.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Heat exchanger conductance and irreversibilities

The heat exchanger conductance is given as

$$\mathrm{UA} = \int_{T_{\mathrm{low}}}^{T_{\mathrm{high}}} \left(\frac{(\dot{m}\cdot c_p)_{\mathrm{H}}}{\Delta T(T_{\mathrm{H}})}\right) \mathrm{d}T_{\mathrm{H}} = (\dot{m}\cdot c_p)_{\mathrm{H}} \cdot \int_{T_{\mathrm{low}}}^{T_{\mathrm{high}}} \left(\frac{1}{\Delta T(T_{\mathrm{H}})}\right) \mathrm{d}T_{\mathrm{H}} \tag{12}$$

where $\Delta T(T_{\mathrm{H}})$ means that ΔT is a function of T_{H}. The irreversibilities associated with the same heat transfer can be found by integrating the difference in exergy of heat between the heat source (hot stream) and the heat sink (cold stream) through the heat exchanger. This can be expressed as

$$\dot{I} = T_0 \cdot \int_0^{\dot{Q}} \left(\frac{1}{T_{\mathrm{C}}} - \frac{1}{T_{\mathrm{H}}}\right) \delta\dot{Q} = (\dot{m}\cdot c_p)_{\mathrm{H}} \cdot T_0 \cdot \int_{T_{\mathrm{low}}}^{T_{\mathrm{high}}} \left(\frac{1}{T_{\mathrm{H}} - \Delta T(T_{\mathrm{H}})} - \frac{1}{T_{\mathrm{H}}}\right) \mathrm{d}T_{\mathrm{H}} \tag{13}$$

where T_0 is the ambient temperature.

Uniform Temperature Difference. For a heat exchanger with a uniform temperature difference $\Delta T = C_{\mathrm{uniform}}$ between the composite curves, the driving forces required to obtain the required heat-transfer conductance UA is given as

$$\Delta T = C_{\mathrm{uniform}} = \frac{(\dot{m}\cdot c_p)_{\mathrm{H}} \cdot (T_{\mathrm{high}} - T_{\mathrm{low}})}{\mathrm{UA}} \tag{14}$$

From Eq. 13, the resulting irreversibilities are given as

$$\dot{I}_{\mathrm{uniform}} = (\dot{m}\cdot c_p)_{\mathrm{H}} \cdot T_0 \cdot \int_{T_{\mathrm{low}}}^{T_{\mathrm{high}}} \left(\frac{1}{T_{\mathrm{H}} - C_{\mathrm{uniform}}} - \frac{1}{T_{\mathrm{H}}}\right) \mathrm{d}T_{\mathrm{H}} = (\dot{m}\cdot c_p)_{\mathrm{H}} \cdot T_0 \cdot \ln\left(\frac{(T_{\mathrm{high}} - C_{\mathrm{uniform}}) \cdot T_{\mathrm{low}}}{(T_{\mathrm{low}} - C_{\mathrm{uniform}}) \cdot T_{\mathrm{high}}}\right) \tag{15}$$

Using Eq. 14, the irreversibilities can be expressed as a function of UA

$$\dot{I}_{\text{uniform}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\ln\left(\frac{\left(T_{\text{high}}-\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)/\text{UA}\right)\cdot T_{\text{low}}}{\left(T_{\text{low}}-\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)/\text{UA}\right)\cdot T_{\text{high}}}\right) \quad (16)$$

Temperature Difference Proportional to Temperature. For the case where the temperature difference is given as a linear function of the absolute temperature, the temperature difference in the hot and cold endpoints of the heat exchanger are given as $\Delta T_{\text{cold}}=C_{\text{linear}}\cdot T_{\text{low}}$ and $\Delta T_{\text{hot}}=C_{\text{linear}}\cdot T_{\text{high}}$, respectively. The heat exchanger conductance can be formulated as

$$\text{UA}=\frac{\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)}{\Delta T_{\text{LM}}} \quad (17)$$

with the logarithmic temperature difference ΔT_{LM} given as

$$\Delta T_{\text{LM}}=\frac{C_{\text{linear}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)}{\ln\left(T_{\text{high}}/T_{\text{low}}\right)} \quad (18)$$

The value of the constant C_{linear} can then be expressed as a function of the heat exchanger conductance

$$C_{\text{linear}}=\frac{\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\ln\left(T_{\text{high}}/T_{\text{low}}\right)}{\text{UA}} \quad (19)$$

With the use of Eq. 13, the irreversibilities associated with the heat transfer can be expressed as

$$\dot{I}_{\text{linear}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\int_{T_{\text{low}}}^{T_{\text{high}}}\left(\frac{1}{(1-C_{\text{linear}})T_{\text{H}}}-\frac{1}{T_{\text{H}}}\right)dT_{\text{H}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\frac{C_{\text{linear}}}{1-C_{\text{linear}}}\times\ln\left(\frac{T_{\text{high}}}{T_{\text{low}}}\right) \quad (20)$$

which by utilizing Eq. 19 can be written as

$$\dot{I}_{\text{linear}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\frac{\left(\ln\left(T_{\text{high}}/T_{\text{low}}\right)\right)^2}{\text{UA}/\left(\dot{m}\cdot c_p\right)_{\text{H}}-\ln\left(T_{\text{high}}/T_{\text{low}}\right)} \quad (21)$$

Constant Difference in Inverse Temperature. When the difference in the inverse of the temperature is constant $\Delta(1/T)=C_{\text{inverse}}$, the heat exchanger temperature difference can be expressed as

$$\Delta T=\frac{C_{\text{inverse}}\cdot T_{\text{H}}^2}{1+C_{\text{inverse}}\cdot T_{\text{H}}} \quad (22)$$

Using Eq. (22), the heat exchanger conductance can be expressed as

$$\text{UA}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\int_{T_{\text{low}}}^{T_{\text{high}}}\left(\frac{1+C_{\text{inverse}}\cdot T_{\text{H}}}{C_{\text{inverse}}\cdot T_{\text{H}}^2}\right)dT_{\text{H}}=\frac{\left(\dot{m}\cdot c_p\right)_{\text{H}}}{C_{\text{inverse}}}\cdot\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}+C_{\text{inverse}}\cdot\ln\left(\frac{T_{\text{high}}}{T_{\text{low}}}\right)\right) \quad (23)$$

Eq. (23) can further be solved for C_{inverse}:

$$C_{\text{inverse}}=\frac{T_{\text{high}}-T_{\text{low}}}{T_{\text{low}}\cdot T_{\text{high}}\cdot\left(\text{UA}/\left(\dot{m}\cdot c_p\right)_{\text{H}}-\ln\left(T_{\text{high}}/T_{\text{low}}\right)\right)} \quad (24)$$

The irreversibilities can be calculated from Eq. 13 as

$$\dot{I}_{\text{inverse}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\int_{T_{\text{low}}}^{T_{\text{high}}}C_{\text{inverse}}dT_{\text{H}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot C_{\text{inverse}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right) \quad (25)$$

which combined with Eq. 24 gives the following result

$$\dot{I}_{\text{inverse}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\frac{\left(T_{\text{high}}-T_{\text{low}}\right)^2}{T_{\text{low}}\cdot T_{\text{high}}\cdot\left(\text{UA}/\left(\dot{m}\cdot c_p\right)_{\text{H}}-\ln\left(T_{\text{high}}/T_{\text{low}}\right)\right)} \quad (26)$$

Temperature Difference Proportional to the Square of the Temperature. For a design with a temperature difference proportional to the square of the temperature of the hot stream, the heat exchanger conductance can be expressed as

$$\text{UA}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\int_{T_{\text{low}}}^{T_{\text{high}}}\left(\frac{1}{C_{\text{square}}\cdot T_{\text{H}}^2}\right)dT_{\text{H}}=\frac{\left(\dot{m}\cdot c_p\right)_{\text{H}}}{C_{\text{square}}}\cdot\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}\right) \quad (27)$$

Hence, the constant C_{square} is given as

$$C_{\text{square}}=\frac{\left(\dot{m}\cdot c_p\right)_{\text{H}}}{\text{UA}}\cdot\left(\frac{1}{T_{\text{low}}}-\frac{1}{T_{\text{high}}}\right) \quad (28)$$

Again, the irreversibilities associated with the heat transfer can be calculated from Eq. 13

$$\dot{I}_{\text{square}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\int_{T_{\text{low}}}^{T_{\text{high}}}\left(\frac{1}{T_{\text{H}}-C_{\text{square}}\cdot T_{\text{H}}^2}-\frac{1}{T_{\text{H}}}\right)dT_{\text{H}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\ln\left(\frac{1-C_{\text{square}}\cdot T_{\text{low}}}{1-C_{\text{square}}\cdot T_{\text{high}}}\right) \quad (29)$$

With the expression from Eq. 28, this can be rewritten as a function of the heat exchanger conductance

$$\dot{I}_{\text{square}}=\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot T_0\cdot\ln\left(\frac{\left(T_{\text{high}}-\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)/UA\right)\cdot T_{\text{low}}}{\left(T_{\text{low}}-\left(\dot{m}\cdot c_p\right)_{\text{H}}\cdot\left(T_{\text{high}}-T_{\text{low}}\right)/UA\right)\cdot T_{\text{high}}}\right) \quad (30)$$

One may observe that the results in Eqs. 16 and 30 are exactly the same. Hence, quite surprisingly, a design with a uniform temperature difference and a design with a temperature difference proportional to the square of the absolute temperature would result in the same total irreversibilities for a given UA value.

Comparison

The difference between the four design guidelines have been illustrated for an example where $T_{\text{low}}=100$ K and $T_{\text{high}}=200$ K. Here, the heat-transfer conductance UA = 50 kW/K and the hot stream heat capacity flow rate $(\dot{m}\cdot c_p)_{\text{H}}=1$ kW/K are given such that the uniform temperature difference required to transfer the heat is equal to 2 K (see Eq. 14). For these calculations, the ambient temperature T_0 was assumed to be 298.15 K.

As can be observed in Table 1, the smallest irreversibilities are obtained for the solution where the temperature difference is given as a linear function of the absolute

Table 1. Heat-Transfer Irreversibilities for Different Heat Exchanger Designs

Design Strategy	Irreversibilities (kW)
1. Uniform temperature difference	3.027
2. Linear function of temperature	2.905
3. Uniform difference in inverse temperature	3.023
4. Quadratic function of temperature	3.027

temperature (design specification 2). As previously discussed, the irreversibilities obtained for a design with a uniform temperature difference and a design where the temperature difference is proportional to the square of the absolute temperature are equal. The irreversibilities in these cases are the largest in the set of four design guidelines. The design where the difference in the inverse of the temperature is constant throughout the heat exchanger provides slightly smaller irreversibilities.

Actually, it was found that a solution with temperature difference $\Delta T = C_1 \cdot T^{1-b}$ gives the same total irreversibilities attributed to heat transfer as a solution with temperature difference $\Delta T = C_2 \cdot T^{1+b}$, where b is any constant. Best performance was then found for $b = 0$, with monotonically increasing irreversibilities for increasing values of b.

The temperature difference throughout the heat exchanger is given as a function of the hot stream temperature for the four different design solutions in Figure 3. As the heat-transfer conductance is inversely proportional to the temperature difference (see Eq. 12) and the UA value is fixed, a larger average temperature difference must be applied in the cases where the temperature difference is reduced in the cold end of the heat exchanger. The designs with $\Delta(1/T) = C_{\text{inverse}}$ and $\Delta T = C_{\text{square}} \cdot T_H^2$ (design specifications 3 and 4) are nearly identical. Again, this can be explained by the fact that for $\Delta T \ll T_{\text{H}}$, $\Delta(1/T) \approx \Delta T/T_{\text{H}}^2$. This also explains why the irreversibilities in these designs are comparable (3.027 kW vs. 3.023 kW).

In Figure 4, the ratio of irreversibility rate to heat-transfer conductance is plotted as a function of the hot stream temperature for the four different design solutions. As can be observed, $\Delta T = C_{\text{linear}} \cdot T_{\text{H}}$ (Design 2) is equivalent to a uniform distribution of irreversibilities per unit of area. This corresponds to the theory of equipartition of entropy production, where the entropy production rate is uniform in time and space.

The ratio of irreversibilities to heat flow is plotted in Figure 5 for the four design guidelines. A design with $\Delta(1/T) = C_{\text{inverse}}$ (Design 3) is equivalent to a uniform distri-

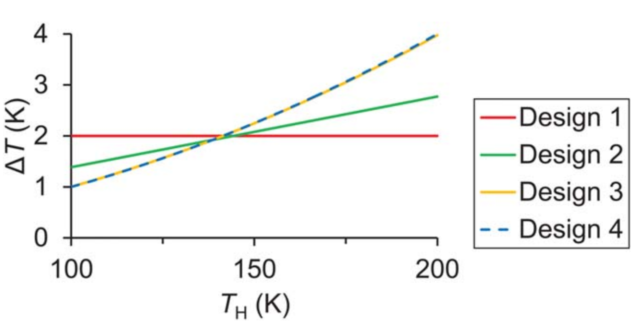

Figure 3. Temperature difference in the heat exchanger as a function of the hot stream temperature.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

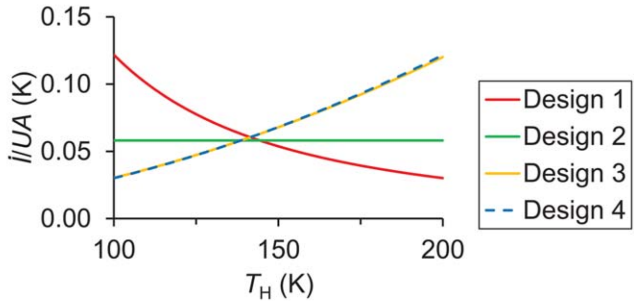

Figure 4. Ratio of irreversibility rate to heat-transfer conductance in the heat exchanger as a function of the hot stream temperature.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

bution of irreversibilities per unit of heat flow. This is illustrated by a horizontal line in Figure 5 and can easily be proven mathematically from Eq. 25. For large values of UA, this is close to true also for the design with $\Delta T = C_{\text{square}} \cdot T_{\text{H}}^2$, as can be observed in Figure 5 where Design 4 has the same horizontal behavior (constant $\dot{I}/\dot{Q}$) as Design 3. When the heat-transfer coefficient is assumed constant, a uniform temperature difference (Design 1) is equivalent to a uniform distribution of heat flow per unit of area, as can be observed from Eqs. 2 and 14.

Influence of operating conditions

For a given heat exchanger size, it was illustrated in the previous sections that a heat exchanger design where the temperature difference between the hot and cold composite curves is given as a linear function of the absolute temperature leads to less exergy destruction than a design with a uniform temperature difference throughout the heat exchanger. To quantify the savings obtainable in such a design, the influence of temperature level, temperature span, and heat exchanger size has been studied.

The savings available when switching from a heat exchanger design with a uniform temperature profile to a design with an optimal temperature profile can be expressed through the ratio of the irreversibilities observed in the former case to the irreversibilities observed in the latter case (this is equivalent to the equipartition factor defined by Thiel et al.[22]). The ratio of irreversibilities in the two cases can be expressed as a function of the low-temperature level of the hot stream T_{low}, the temperature span of the hot stream

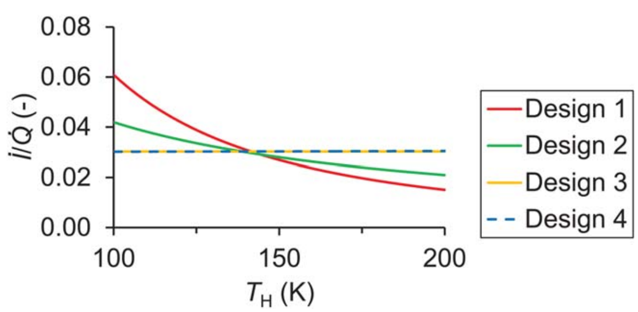

Figure 5. Ratio of irreversibility rate to heat flow in the heat exchanger as a function of the hot stream temperature.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

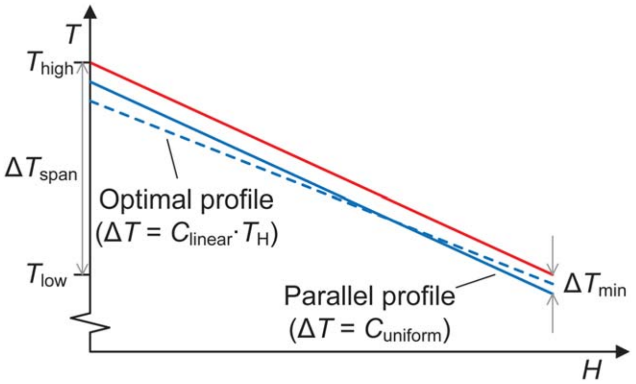

Figure 6. Composite curves for a simple heat exchanger.

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$\Delta T_{\text{span}} = T_{\text{high}} - T_{\text{low}}$, and the minimum temperature difference $\Delta T_{\min} = C_{\text{uniform}}$

$$\frac{\dot{I}_{\text{linear}}}{\dot{I}_{\text{uniform}}} = \frac{\left(\ln\left(\left(T_{\text{low}} + \Delta T_{\text{span}}\right)/T_{\text{low}}\right)\right)^2}{\left(\frac{\Delta T_{\text{span}}}{\Delta T_{\min}} - \ln\left(\frac{T_{\text{low}} + \Delta T_{\text{span}}}{T_{\text{low}}}\right)\right) \cdot \ln\left(\frac{\left(T_{\text{low}} + \Delta T_{\text{span}} - \Delta T_{\min}\right) \cdot T_{\text{low}}}{\left(T_{\text{low}} - \Delta T_{\min}\right) \cdot \left(T_{\text{low}} + \Delta T_{\text{span}}\right)}\right)} \tag{31}$$

The minimum temperature difference is used as a parameter defining the heat exchanger size rather than the heat exchanger conductance as this is a property that more easily relates to practical experience. The heat exchanger conductance UA is still the same for both designs, yet expressed indirectly through the equivalent uniform temperature difference calculated from Eq. 14. Here, the problem is defined such that with variation in the temperature span for the hot stream, the ratio of heat-transfer conductance to total heat flow is constant, rather than the total heat-transfer conductance. The difference between the two formulations (uniform vs. linear) is illustrated in a temperature-enthalpy diagram in Figure 6.

In Figure 7, the ratio of irreversibilities obtained with $\Delta T = C_{\text{linear}} \cdot T_{\text{H}}$ to irreversibilities obtained with $\Delta T = C_{\text{uniform}}$ is plotted as a function of the hot stream target temperature T_{low}. This is done for different values of temperature span for the hot stream ΔT_{span}, with a UA value equivalent to $\Delta T_{\min} = 2$ K. For large values of the target temperature, savings observed for the optimal temperature profile are relatively small as the ratio of irreversibilities approaches unity with increasing temperatures. For small values of the target temperature, however, significant savings are observed for a design where the temperature difference is proportional to the absolute temperature. This can be explained by the fact that the exergy of heat grows steeply with decreasing temperature below ambient, as illustrated in Figure 1, or more accurately because the slope of the derivative with respect to temperature increases with decreasing temperature. Thus, in sub-ambient processes such as liquefaction of natural gas, optimal distribution of driving forces could provide significant energy savings.

As can be observed in Figure 8, the savings obtained for the optimal temperature profile relative to a uniform temperature difference increase with increasing temperature span for the hot stream. With increasing temperature span, the relative difference in exergy of heat increases. Hence, the influence of the distribution of driving forces increases. For the same temperature span, the difference in exergy of heat between the supply and target temperatures increases with decreasing absolute temperature.

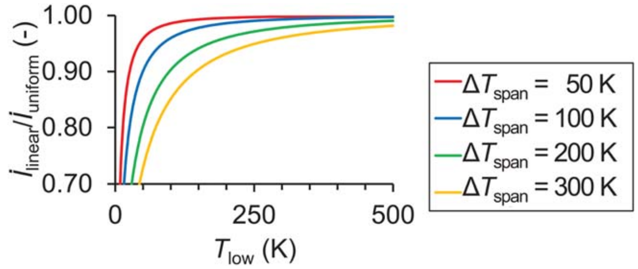

Figure 7. Ratio between irreversibilities in a heat exchanger for the optimal temperature profile to irreversibilities with a uniform temperature difference as a function of the hot stream target temperature for different values of temperature span and with a UA value corresponding to $\Delta T_{\min} = 2$ K.

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The influence of the minimum temperature difference (or indirectly the heat-transfer conductance) is also illustrated in Figure 8, where the ratio of irreversibilities obtained with $\Delta T = C_{\text{linear}} \cdot T_{\text{H}}$ to irreversibilities obtained with $\Delta T = C_{\text{uniform}}$ is plotted as a function of the temperature span. This is done for different values of the minimum temperature difference, with a hot stream target temperature $T_{\text{low}} = 100$ K. The results indicate that the size of the heat exchanger is of little influence with respect to the potential savings with an optimal temperature profile. Only for large values of the temperature span and/or low target temperatures, increased savings from using the optimal temperature profile are observed for increasing value of the temperature difference (smaller heat exchangers).

At relatively high-temperature levels, for cooling loads distributed over a relatively narrow temperature span, the energy penalty associated with a uniform temperature difference ΔT is quite small. Hence, for many heat exchanger network design problems encountered in industry, optimal distribution of temperature driving forces with respect to

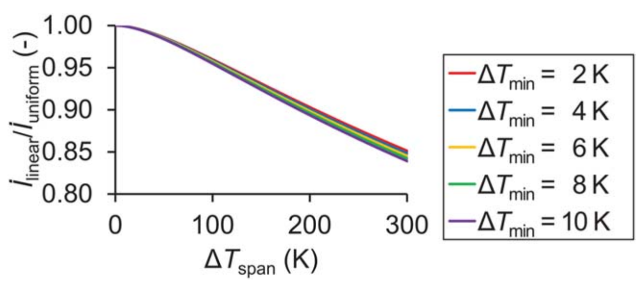

Figure 8. Ratio between irreversibilities in a heat exchanger for the optimal temperature profile and a uniform temperature difference as a function of the temperature span for different values of the minimum temperature difference and hot stream target temperature $T_{\text{low}} = 100$ K.

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exergy of heat is of little importance, confirming the success of the pinch design method. This also explains why little difference in entropy production was observed between designs with equipartition of entropy production, equipartition of forces, and uniform temperature difference in the studies presented by Balkan.[15]

In cryogenic applications where cooling is provided at subambient temperature and typically over a wide temperature span, the results do indicate that a design with a uniform temperature difference throughout the heat-transfer process leads to considerable penalties in energy use. This suggests that a minimum temperature difference is an inadequate economic trade-off parameter for design of cryogenic processes. This also explains why Jensen and Skogestad[4] found a minimum temperature difference requirement to give nonoptimal utilization of the heat exchanger area when designing a process for liquefaction of natural gas.

Conclusions

In this work, four different design guidelines have been compared for optimal distribution of temperature driving forces in heat exchangers. This was done for a simple heat exchanger model assuming constant heat-transfer coefficient. The results confirmed that the irreversibilities in a heat exchanger of given size are minimized when the temperature difference is proportional to the temperature at which heat is transferred, which is equivalent to a uniform distribution of entropy production per unit of area. The common design guideline based on a uniform temperature difference leads to nonoptimal utilization of the heat exchanger area and thereby increased irreversibilities.

Sensitivity analyses were conducted to compare the performance of a design with uniform temperature difference throughout the heat exchanger and a design with optimal distribution of temperature driving forces. The results indicate that the savings in irreversibilities with optimal design increase with decreasing temperature level and increasing temperature span for the cooling load. This is related to the exergy of heat as function of temperature, for which the steepness of the derivative increases with decreasing temperature.

The performance of a heat exchanger with respect distribution of driving forces can be evaluated by the ratio between the irreversibilities in a heat exchanger with optimal distribution and the irreversibilities in the actual heat exchanger (given that the two heat exchangers have the same size and the same hot composite curve, or alternatively the same cold composite curve). As previously discussed, the former is obtained when the temperature driving forces are proportional to the temperature level. This measure is equivalent to the equipartition factor proposed in literature.

The results obtained in this work indicate that a design criterion based on a minimum temperature difference constraint will provide good performance in systems operating above ambient temperature, which is encountered in many heat exchanger network design problems. For processes operating at low-temperature level and over wide temperature span, conversely, these results indicate that a design with uniform temperature difference would lead to considerable penalty in energy use. This suggests that a minimum temperature difference constraint would lead to nonoptimal utilization of heat exchanger area in processes for natural gas liquefaction, hydrogen liquefaction, olefin plants, and other cryogenic applications, indicating that a minimum temperature difference is an inadequate economic trade-off parameter for design of low-temperature processes.

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Notation

Greek letters

Δ = difference
$\dot{\varepsilon}$ = exergy flow rate, kW
ψ = rational exergy efficiency, −

Roman letters

A = area, m^2
a = heat exchanger unit cost, USD/m^2
b = constant
C = constant
c = constant
c_p = specific heat capacity, kJ/kg K
e = exergy unit cost, USD/kW
I = irreversibility rate, kW
k = constant
L = phenomenological heat-transfer coefficient, kW K/m^2
$\dot{m}$ = mass flow rate, kg/s
$\dot{Q}$ = heat flow rate, kW
$\dot{q}$ = heat flux, kW/m^2
$\dot{S}$ = entropy flow rate, kW
T = temperature, K
U = overall heat-transfer coefficient, kW/m^2K
UA = heat exchanger conductance, kW/K

Subscript

0 = ambient
C = cold
H = hot
high = supply
i = point i in heat exchanger
in = in
inverse = with a constant temperature driving force $\Delta(1/T)$
linear = with a temperature difference proportional to the absolute temperature
LM = logarithmic mean
low = target
out = out
prod = produced
span = difference between supply and target
square = with a temperature difference proportional to the square of the absolute temperature
uniform = with a uniform temperature difference ΔT

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